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# SURFACE PHENOMENA IN THE HYDRODYNAMICS OF AN INCOMPRESSIBLE FLUID

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*By taking as an example the outflow of an incompressible fluid through a hole with small geometric parameters under the conditions of supersmall pressures, we attempted to evaluate the effect of surface phenomena on its flow by the similarity method. The Navier–Stokes differential equation was supplemented by parameters allowing for the effect of surface forces. The modified Navier–Stokes equation was subjected to similarity conversion. This yielded a dimensionless group that includes the whole range of variable parameters affecting the fluid flow, namely, the generalized criterion Pv. The graphic dependence of the coefficient of the fluid flow rate on the generalized criterion Pv is presented on the basis of experimental data.*

Underlying surface phenomena are intermolecular interactions which are not taken into account in solving hydrodynamic problems by elementary hydraulics. However, the effect of surface phenomena on the hydrodynamics of viscous incompressible fluid flow increases greatly in organizing fluid film flow under gravity in technological petrochemical and petroleum-refining equipment. Investigation of fluid outflow through the holes of low-pressure (when $h < 0.2$ m $H_2O$) jet and slot distributors of fluid in packed columns and film devices revealed the increase in the value of the coefficient of fluid flow rate $\mu$ with decrease in pressure (decrease of the fluid level over the distributing plate) [1, 2]. In our opinion, the main reason for the change in the energy of the jet on outflow through the hole is the effect of the surface phenomena appearing in interaction of surface forces at the phase interfaces of the systems: "fluid–solid," "liquid–gas," and "solid–gas." At low pressures when the mass forces of the fluid flow become commensurable to surface forces, the latter begin to considerably affect the hydrodynamic characteristics of the fluid flow escaping from a hole. The mechanisms of the effect of surface forces on fluid flow are based on such surface phenomena [3] as the generation of the Gibbs surface energy as a result of the formation of a free surface of the fluid on outflow of a jet from the hole due to the decrease of its energy; adhesion (sticking) of the fluid to the hole walls; spreading of outflowing fluid over the bottom surface, adjacent to the hole, and wetting it; capillary phenomena resulting from the formation of a meniscus between the jet and the fluid impinging on the bottom; setting-up of additional pressure by surface tension forces in the jet with the surface curvature; moreover, with small diameters of holes and supersmall pressures, hanging of the fluid in the channel is observed. The hanging occurs at the beginning of outflow when a meniscus with a positive curvature is formed in the fluid outflowing from the hole. The developing Laplace pressure opposes the fluid flow. The fluid hangs until the statistical pressure of the fluid exceeds the Laplace pressure in magnitude.

The coefficient of flow rate in fluid outflow from a conoidal hole ($\varepsilon = 1$), other conditions being equal, changes due to pressure increment, which is a result of the effect of surface forces at the phase interfaces. The value of the resultant of these forces affects the hydrodynamics of the flow. We are familiar with the "Thoms effect," which is the reduction of hydraulic resistance in fluid flow in a tube after introducing

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small amounts of polymer additives which are likely to play the role of surfactants [4]. It is noted that the additives which decrease hydraulic resistance were injected either into the near-wall or central part of the flow. In the first case, the hydraulic resistance decreased instantly, whereas in the second case, the hydraulic resistance decreased only after the additives had diffused into the near-wall region. The introduction of additives in such small amounts actually does not change the density and viscosity of fluid and must not influence noticeably the distribution of velocities in the flow, but it can affect the character of the near-wall flow, i.e., the boundary conditions on the interfaces between the bulk of the moving fluid and the motionless channel walls change. It was noted in [5] that "... in this case, the adopted Navier–Stokes theory of viscous fluid flow requires substantial revision."

Taking into account the foregoing, we introduce into the Navier–Stokes equation terms reflecting the effect of surface forces and write this equation in the vector form

$$F_{\mathbf{i}} + F_{\mathbf{g}} + F_p + F_{\boldsymbol{\eta}} + F_{\boldsymbol{\sigma}} = 0\ .$$

We employ the method of hydrodynamic similarity [6], for which purpose we consider separate terms of the equation with allowance for the effect of surface forces.

We rewrite the Navier–Stokes equation for a dropping liquid in an expanded form for one of the axes — the vertical $z$ axis — with account for the surface tension forces of the participating phases:

$$\rho\left(\frac{\partial v_z}{\partial \tau} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = -\rho g - \frac{\partial p}{\partial z} + \eta\vartheta\left(\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right) + \sigma\frac{\partial A_{\text{sp}}}{\partial z}\ .$$

We investigate the effect of surface forces on the magnitude of pressure in the liquid jet escaping from a hole. Under the conditions adopted, the pressure above the liquid and that under the hole are the same; moreover, owing to the conoidal profile of the hole ($\varepsilon$ = 1) there is no annular isolated cavity with vacuum near its walls. The jet of liquid escaping from the slotted hole into the gas has a surface curvature radius $r = \infty$. Here, the surface forces of the bottom adjacent to the hole affect the surface liquid layers of the jet by the force of spreading [7], which for a smooth homogeneous surface is $\Delta\sigma = \sigma(\cos\theta - \cos\theta_{\text{dyn}})$.

When the liquid outflows from a hole, the dynamic wetting angle is $\theta_{\text{dyn}} = 90^{\circ}$, then $\Delta\sigma = \sigma\cos\theta$.

The force of spreading related to unit length of the wetting perimeter is directed normally from the jet surface; it decreases the pressure in it by the value $\Delta p_1$. It should be noted that the radius of the meniscus formed between the jet and the liquid flowing onto the bottom is related to the magnitude of this force as

$$\Delta p_1 = \frac{\Delta\sigma\Pi v}{V_{\text{v}}} = \frac{\Delta\sigma\Pi}{f} = \frac{4\sigma\cos\theta}{d_{\text{eq}}}\ .$$

Subjecting the modified Navier–Stokes equation to similarity conversion, we obtain the similarity criteria by dividing one part of the differential equation by the other with subsequent omission of the signs of the mathematical operators.

The ratio between the pressure increment and the inertia forces represents the Euler criterion under the conditions of the effect of surface forces:

$$\text{Eu}_\sigma = \frac{F_p}{F_{\text{i}}} = \frac{\Delta p/l}{\frac{\rho v^2}{l}} = \frac{4\sigma\cos\theta}{d_{\text{eq}}\rho v^2} = \frac{4\sigma\cos\theta}{2gh\rho d_{\text{eq}}}\ ; \quad \text{Eu}_\sigma = \frac{4\cos\theta}{\text{We}}\ .$$

We consider the effect of surface forces on the pressure in the jet the surface of which has the curvature radius. For this purpose, we take the simplest case of a jet escaping from a round hole of a conoidal profile ($d_j = d_h$). The pressure loss due to compression of the jet by surface tension forces in its escape from the round hole is [8]

$$\Delta h = \frac{2\sigma}{\rho g d_j},$$

whence the pressure increment is $\Delta p_2 = 2\sigma/d_h$.

The total change in the pressure in the round jet escaping from the hole into the gas, with account for the effect of the force of spreading, is

$$\Delta p = \Delta p_1 - \Delta p_2 = \frac{4\sigma\cos\theta}{d_h} - \frac{2\sigma}{d_h} = \frac{2\sigma\,(2\cos\theta - 1)}{d_h}.$$

In this case, the Euler criterion with account for the effect of surface phenomena acquires the form $Eu_\sigma = 2(2\cos\theta - 1)/We$.

We consider the term which in the equation determines the magnitude of the forces of internal friction in the flow of liquid. According to the Newton law of internal friction, the internal friction stress arising between the layers of the moving liquid is directly proportional to the velocity gradient. The latter is determined by the boundary conditions. Usually, for a liquid outflowing from a hole it is assumed a priori that at the hole walls the liquid has a zero velocity, i.e., its near-wall layer adheres emtirely to the walls. This becomes possible only if the adhesion forces exceed in magnitude the cohesion forces, but if the surface tension force of the liquid exceeds the tension force of the solid body (hole walls), then the adherence is only partial. This gives the possibility for the liquid to slip along the walls.

We represent the change in the boundary conditions at the boundaries of the moving medium by introducing the sticking coefficient $\vartheta$ into the equation. Then the term accounting in the Navier–Stokes equation for the effect of the friction force takes the form

$$F_\eta = \eta\vartheta\left(\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right) \sim \frac{\eta\vartheta v}{l^2}.$$

The ratio of the inertia forces to the friction forces with account for surface phenomena gives the similarity criterion

$$Re_\sigma = \frac{F_i}{F_\eta} = \frac{\rho v^2/l}{\eta\vartheta v/l^2} = \frac{\rho v d_{eq}}{\eta\vartheta},$$

and, since $\vartheta = (1 + \cos\theta)/2$

$$Re_\sigma = \frac{2\,Re}{(1 + \cos\theta)}.$$

In characterizing the Reynolds number, it was noted in [9] that the numerical value of Re was not the quantity strictly proportional to the inertia-to-friction force ratio.

When adherence is incomplete, the near-wall layers of liquid slip through. The value of the velocity gradient decreases. The value of jet energy loss to overcome the internal friction decreases. The reduction of the hydraulic losses causes an increase in the flow velocity which, in turn, increases the Re number. Simultaneously, the constancy of the liquid viscosity continues to counteract disturbances with the same force. As a result, the critical Re number increases, contributing to the stability of the laminar regime of the flow of liquid.

We consider the last term of the modified Navier–Stokes equation; it determines the surface tension force that influences the flow of the liquid escaping from the hole into a gaseous medium. As the liquid flows out of the hole, a free surface of the liquid is formed whose area depends on the length of the hole perimeter and the liquid flow velocity. Thus at the expense of the jet energy the Gibbs free surface energy is formed which is proportional to the area of the formed free surface of the liquid:

$$F_\sigma = \frac{\sigma \Pi v}{V_v l} = \frac{\sigma \Pi}{fl} = \frac{4\sigma}{d_{eq} l}.$$

As is known, the ratio between the inertia forces and surface tension forces is the Weber criterion. Under the conditions of the effect of surface phenomena, the Weber criterion for the liquid outflowing from a hole takes the form

$$\mathrm{We}_\sigma = \frac{F_i}{F_\sigma} = \frac{\rho v^2/l}{4\sigma/d_{eq} l} = \frac{\rho v^2 d_{eq}}{4\sigma} = \frac{\mathrm{We}}{4}; \quad \mathrm{We}_\sigma = 0.25\ \mathrm{We}.$$

It is seen from the foregoing analysis that the flow of incompressible viscous liquid through a conoidal hole in a thick wall is affected by the forces arising as a result of surface interaction of the participating phases. Their effect extends to the forces of pressure $F_p$, viscosity $F_\eta$, and surface tension of the liquid $F_\sigma$. We find the resultant of these forces: $F_p + F_\eta + F_\sigma = F$. Having substituted the values of the terms for a slotted hole into the equation, we obtain

$$F = -\frac{4\sigma \cos\theta}{l^2} + \frac{\eta v \vartheta}{l^2} + \frac{4\sigma}{l^2} = \frac{4\sigma(1-\cos\theta) + \eta v \vartheta}{l^2}.$$

The ratio of the inertia forces $F_i$ to the resultant of the forces that experience the effect of the surface phenomena arising in the liquid outflowing through the hole into the gas represents the similarity criterion which involves the entire range of variable parameters affecting the liquid flow:

$$\mathrm{Pv} = \frac{F_i}{F} = \frac{\rho v^2/l}{\dfrac{4\sigma(1-\cos\theta) + \eta v \vartheta}{l^2}} = \frac{\rho v^2/l}{4\sigma(1-\cos\theta) + \eta v \vartheta}.$$

In order to simplify this expression, we consider the reciprocal quantity

$$\frac{1}{\mathrm{Pv}} = \frac{4\sigma(1-\cos\theta) + \eta v \vartheta}{\rho v^2 l} = \frac{4\sigma(1-\cos\theta)}{\rho v^2 l} + \frac{\eta v \vartheta}{\rho v^2 l} = \frac{4(1-\cos\theta)}{\mathrm{We}} + \frac{\vartheta}{\mathrm{Re}},$$

and since $\vartheta = (1 + \cos\theta)/2$

$$\frac{1}{\mathrm{Pv}} = \frac{8\,\mathrm{Re}(1-\cos\theta) + \mathrm{We}(1+\cos\theta)}{2\,\mathrm{We}\,\mathrm{Re}} = \frac{4\,\mathrm{Re}(1-\cos\theta) + 0.5\,\mathrm{We}(1+\cos\theta)}{\mathrm{We}\,\mathrm{Re}}.$$

Then

$$\mathrm{Pv} = \frac{\mathrm{We}\,\mathrm{Re}}{4\,\mathrm{Re}(1-\cos\theta) + 0.5\,\mathrm{We}(1+\cos\theta)}.$$

Similar derivation of the criterion for a jet escaping from a conoidal round hole yields

$$\mathrm{Pv} = \frac{\mathrm{We\,Re}}{2\,\mathrm{Re}\,(3 - 2\cos\theta) + 0.5\,\mathrm{We}\,(1 + \cos\theta)}\,.$$

The similarity criterion Pv is the combination of the criteria Re and We with account for the effect of intermolecular interactions arising at the phase interfaces of the participating phases. It allows for the entire range of variable quantities, which in combination affect the process of liquid outflow and make it possible to study an infinite set of different cases rather than a single particular case. The use of the Pv criterion imparts a general character to the analysis and creates the possibility of refined estimation of experimental results.

In the course of experimental investigation, the effect of surface phenomena on the hydrodynamics of incompressible viscous liquid flow in the channel was estimated by a change in the value of the coefficient of flow rate $\mu$. Since the holes had a conoidal profile (constructed by the portion of the lemniscate), for which $\varepsilon = 1$, the coefficient of flow rate $\mu$ is equal to the coefficient of velocity $\varphi$ ($\mu = \varphi$). The latter, in turn, reflects a change in flow velocity as a function of the total hydraulic losses caused by the effect of variable parameters, including the surface phenomena. The functional dependence of the coefficient of the flow rate $\mu$ of an incompressible fluid through a small hole with smooth walls in a large open reservoir on variable quantities has the form $\mu = f(l, d_{\mathrm{h}}, \gamma, d_{\mathrm{col}}, \Pi, h, \Delta p, \rho, \theta, \sigma, \eta, g)$. After the introduction of the geometric similarity criteria and the criteria Pv and Fr, the function takes the form $\mu = f(i_1, \mathrm{Pv}, \mathrm{Fr})$.

Having introduced fixed values of $i_1$ and Fr into the conditions of the experiment, we can determine the dependence $\mu = f(\mathrm{Pv})$. The geometric parameters are kept constant due to the studied holes being of the same length $l = 4$ mm and also the conoidal profile with the inflow angle $\gamma$ equal to zero [10]. The simplex $l/d$ has different values depending on the equivalent diameter $d_{\mathrm{eq}}$ taken, but in the absence of a vacuum annular cavity in the initial section of the hole ($\varepsilon = 1$) and owing to the small length of the hole $l$ the effect of its change can be neglected. The simplex $d_{\mathrm{col}}/d_{\mathrm{h}}$ characterizes separationless inflow, which is ensured by the conoidal profile of the hole.

The studied processes produced in the experiment have the values $\mathrm{Fr} > 10$ at which self-similarity is observed [8]. In this way, the ambiguity conditions with respect to the criterion Pv are met.

The method of experimental investigation makes provision for carrying out successively a series of experiments in which all the following quantities entering into the criterion Pv change in wide ranges: the velocities of outflow due to the change in the magnitude of pressure $h$; the physical properties of the hole walls due to the use of different materials from which they are made, the physical properties of the liquid due to the study of different liquids at different temperatures. The geometric parameters of the hole studied — shape, equivalent diameter, and the perimeter of the hole walls — were changed; the parameter of the intermolecular interaction of phases at the phase interface (wetting angle $\theta$) changed owing to the use of different combinations of materials in the system. The experiment was performed on a pilot setup with replaceable elements with different types of holes of the third accuracy rating and sixth surface roughness grade. The pressure tank with a discharge device ensured a constant pressure $h$, which was changed from 0.02 to 0.2 m $H_2O$. In the experiment, theoretical liquid flow rate through the hole was determined by calculation and the actual flow rate of liquid under certain conditions was found experimentally. The results obtained gave the flow rate coefficient $\mu$. The experimental investigation carried out gave sufficient tabulated data to construct the graph of the dependence of the flow-rate coefficient $\mu$ on the value of the criterion Pv (see Fig. 1).

An analysis of the dependence $\mu = f(\mathrm{Pv})$ shows that the complex effect of surface phenomena facilitates reduction in total hydraulic losses resulting in an increase in the flow-rate coefficient $\mu$. The maximum positive effect of these losses manifests itself at Pv = 15–40 when the value of the flow-rate coefficient reaches unity, with some experiments showing $\mu > 1$. As the criterion Pv increases further, the effect of sur-

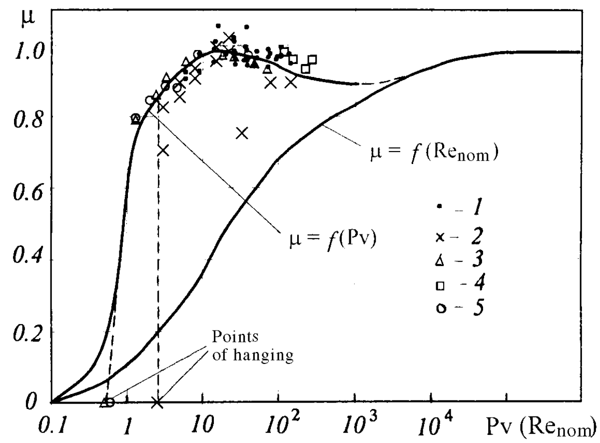


Fig. 1. Coefficient of fluid flow rate μ through conoidal holes (ε = 1) as a function of Pv and $Re_{nom}$ (see Table 1).

TABLE 1.

| No. in Fig. 1 | Characteristics of condensed phases | | Wetting angle, $\theta^o$ | Characteristics of the hole | | |
|---|---|---|---|---|---|---|
| | wall material | fluid | | shape | diameter, mm | perimeter, mm |
| 1 | Stainless steel | Water, $\rho = 998$ kg/m$^3$, $\sigma = 72\cdot10^{-3}$ N/m, $\nu = 0.94\cdot10^{-6}$, $1.1\cdot10^{-6}$ m$^2$/sec | 42.5 | Round<br>Slot | 8–10<br>0.6–3.7 | 25–31<br>54–336 |
| 2 | Stainless steel | Vacuum solar oil, $\rho = 855.4$ kg/m$^3$, $\sigma = 32.4\cdot10^{-3}$ N/m, $\nu = 10.62\cdot10^{-6}$ m$^2$/sec | 29 | Round | 8 | 25 |
| 3 | Polyethy-lene | Water* | 71 | Slot | 0.6–4.7 | 75–201 |
| 4 | Stainless steel | Lighting kerosene, $\rho = 778$ kg/m$^3$, $\sigma = 27.1\cdot10^{-3}$ N/m, $\nu = 1.93\cdot10^{-6}$ m$^2$/sec | 0 | Round | 8–10.1 | 25; 31 |
| 5 | Fluoro-plastic | Water* | 108 | Slot<br>Round | 0.6; 1.6<br>1.0; 3.0 | 121; 162<br>3; 9 |

* Water properties correspond to experiment No. 1.

face phenomena on the flow-rate coefficient decreases. In the figure, the points of liquid hanging in the hole are plotted. The experiments showed that with water-repellent surfaces of the hole (fluoroplastic, polyethylene) the pressure of hanging $h_{hang}$ is low, which corresponds to Pv = 0.5–1, whereas with hydrophilic surfaces the pressure of hanging increases and, correspondingly, Pv = 2.5–3. As the static pressure exceeds the value of the pressure of hanging ($h > h_{hang}$), the liquid begins to flow and the coefficient of flow rate increases sharply from zero to its nominal value. In the figure it is shown by dashed lines. It should be noted that the hanging effect is observed only when the hole walls are dry (the force of surface tension of a solid body at the interface with the gas is maximum). As the walls are covered by a liquid film, the phenomenon of autophobia appears and the liquid drains off the hole completely. Hanging is not observed, since a meniscus with positive curvature is not formed.

In addition to the dependence $\mu = f(\mathrm{Pv})$, Fig. 1 presents the curve of the dependence $\mu = f(\mathrm{Re}_{nom})$ for the hole of conoidal profile; the curve is constructed on the basis of Al'tshul's graphs [10] and it does not allow for the effect of surface phenomena. Comparison of these two curves allows one to estimate quantitatively the effect of surface phenomena on incompressible liquid flow through the hole.

Using the obtained dependence $\mu = f(\mathrm{Pv})$, after the introduction of corrections [10], for other values of $i_1$ and Fr that affect the liquid flow, it is possible to obtain the value of μ for other conditions of liquid outflow.

Thus, based on the analysis of the physical mechanism of the studied process and similarity conversion of the Navier–Stokes equation, we have determined the structure of the dimensionless complex Pv which characterizes the process of fluid flow through holes with account for arising surface phenomena. An experimental investigation was made on models, and the effect of surface phenomena on fluid flow through holes was determined.

This work is of applied importance for solving problems on selecting materials and calculation of hydrodynamic characteristics of fluid outflow in creation of highly efficient designs of low-pressure film devices.

## NOTATION

$F_i$, inertia forces, N; $F_g$, gravity forces, N; $F_p$, pressure forces, N; $F_\eta$, friction forces, N; $F_\sigma$, surface tension forces of outflowing fluid, N; $h$, fluid pressure, m; $\mu$, coefficient of fluid flow rate; $x$, $y$, and $z$, coordinate axes; $\rho$, density, kg/m$^3$; $g$, free-fall acceleration, m/sec$^2$; $p$, pressure, Pa; $\eta$, dynamic viscosity, Pa·sec; $v_x$, $v_y$, and $v_z$, velocity components, m/sec; $\tau$, time, sec; $\sigma$, surface tension at the liquid–gas interface, N/m; $A_{sp}$, specific surface of the outflowing medium, m$^2$/m$^3$, $A_{sp} = A/V$; $A$, area of the surface of the outflowing medium, m$^2$; $V$, volume, m$^3$; $\vartheta$, coefficient of sticking; $\vartheta = W_a/W_{coh} = (1 + \cos\theta)/2$ [3]; $W_a$, work of adhesion; $W_{coh}$, work of cohesion; $\theta$, wetting angle, deg; $\theta_{dyn}$, dynamic wetting angle, deg; $\varepsilon$, coefficient of jet compression; $r$, radius of curvature of the jet surface, m; $d_{eq} = 4f/\Pi$, equivalent diameter, m; $f$, flow area of the hole, m$^2$; $\Pi$, wetted perimeter, m; $l$, linear size, hole length, m; $V_v$, volumetric flow rate of fluid, m$^3$/sec; Eu, Euler criterion; $v$, velocity, m/sec; We, Weber criterion; $d_j$, jet diameter, m; $d_h$, hole diameter, m; Re, Reynolds criterion; $Re_\sigma$, Reynolds criterion with account for forces of surface interaction; $We_\sigma$, Weber criterion with account for forces of surface interface; $F$, resultant, N; Pv, generalizing criterion; $\varphi$, coefficient of velocity; Fr, Froude criterion; $\gamma$, inflow angle, deg; $d_{col}$, collector diameter, m; $i_1$, parametric criteria ($l/d_h$, $\gamma$, $d_{col}/d_h$); $Re_{nom}$, approximate Reynolds criterion; $h_{hang}$, hanging pressure, m; $\nu$, kinematic viscosity, m$^2$/sec. Subscripts: i, inertia; g, gravitation; $p$, pressure; $\eta$, viscosity; $\sigma$, surface tension; $x$, $y$, $z$, coordinate axes; sp, specific surface; a, adhesion; coh, cohesion; dyn, dynamic; eq, equivalent; j, jet; h, hole; v, volumetric; col, collector; 1, geometric parameters; hang, hanging; nom, nominal.

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